Lomonosov Moscow
State University

Quasiparticle self-recombination in Superconducting Tunnel Junction X-ray detectors

V.A. Andrianov[1], V.P. Gorkov[2]

[1] Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University,
[2] Faculty of Computational Mathematics and Cybernetics, Lomonosov Moscow State University,
Moscow, Russia, e-mail: andrva22@mail.ru

1. Motivation

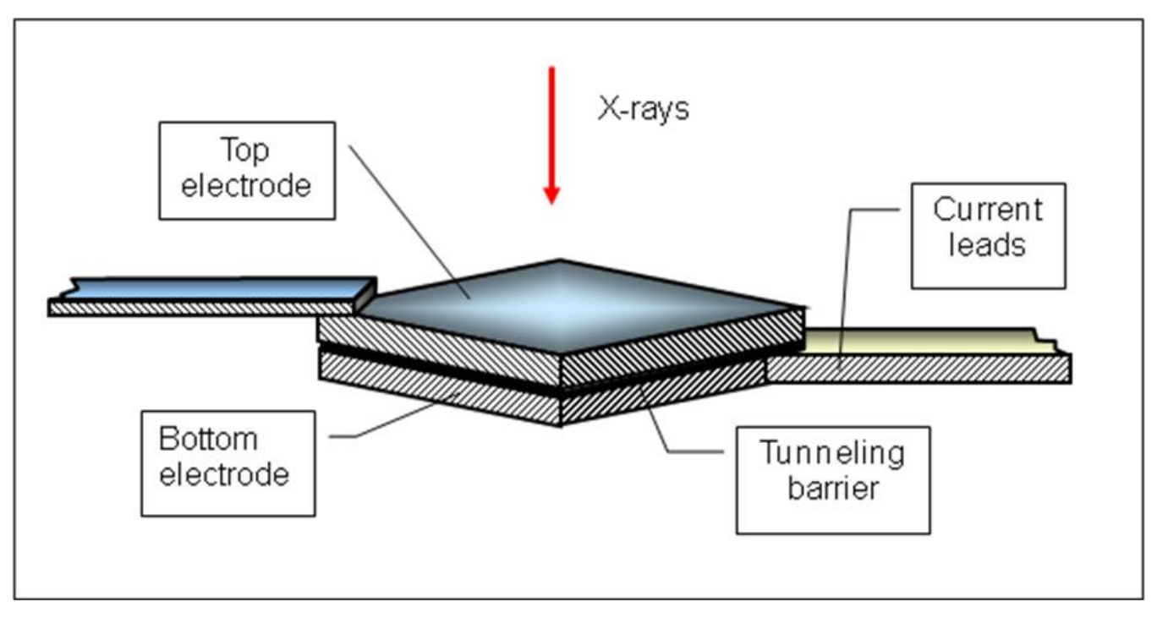
Detectors based on superconducting tunnel junctions (STJ) have high energy resolution and low energy threshold and can be used in the precision X-ray, ultraviolet and optical spectroscopy. The advantage of STJ-detectors compared with the calorimetric detectors is faster response that may be from 0.1 to 10 microseconds [1]. Unfortunately, in soft X-ray range the energy resolution is noticeably worse than the theoretical predictions.

In this work, the mathematical modeling of STJ-detectors was performed on base of the diffusion equation for nonequilibrium quasiparticles, and taking into account tunneling, loss of quasiparticles, self-recombination and exchange 2Δ-phonons. The effect of self-recombination is examined in detail. It was shown that the self-recombination causes 1) a nonlinear dependence of the detector response on the photon energy; 2) broadening of the spectral line; and 3) changing in the temporal shape of the detector signals.

The conditions of the compensation of recombination losses were considered. A simple analytical expression was obtained. Noticeable weakening of recombination effects is expected in cases when the compensation condition was satisfied for the electrode where the photon has been absorbed. In particular, the dependence of the signal amplitude on the photon energy is almost linear, and recombination broadening of the detector line is significantly weakened.

[1] P. Lerch, A. Zender, A., *Top. in Appl. Phys.*, Ed. C. Enss (2005) 217–265.
[2] V.A. Andrianov and et al.NIM A, **559** (2006) p.683
[3] V.A. Andrianov, L.V. Filippenko J. Low Temp. Phys. **167** (2012) 404.

Scheme of STJ-detector



3. Detectors with one active Electrode

Most strongly the recombination manifests itself in STJ detectors with one active electrode.

Figure 1. The dependence of the signal *Q* on the energy of absorbed photons with recombination coefficients R_1 = 100 and R_2 = 200 (curves *1* and *2*).

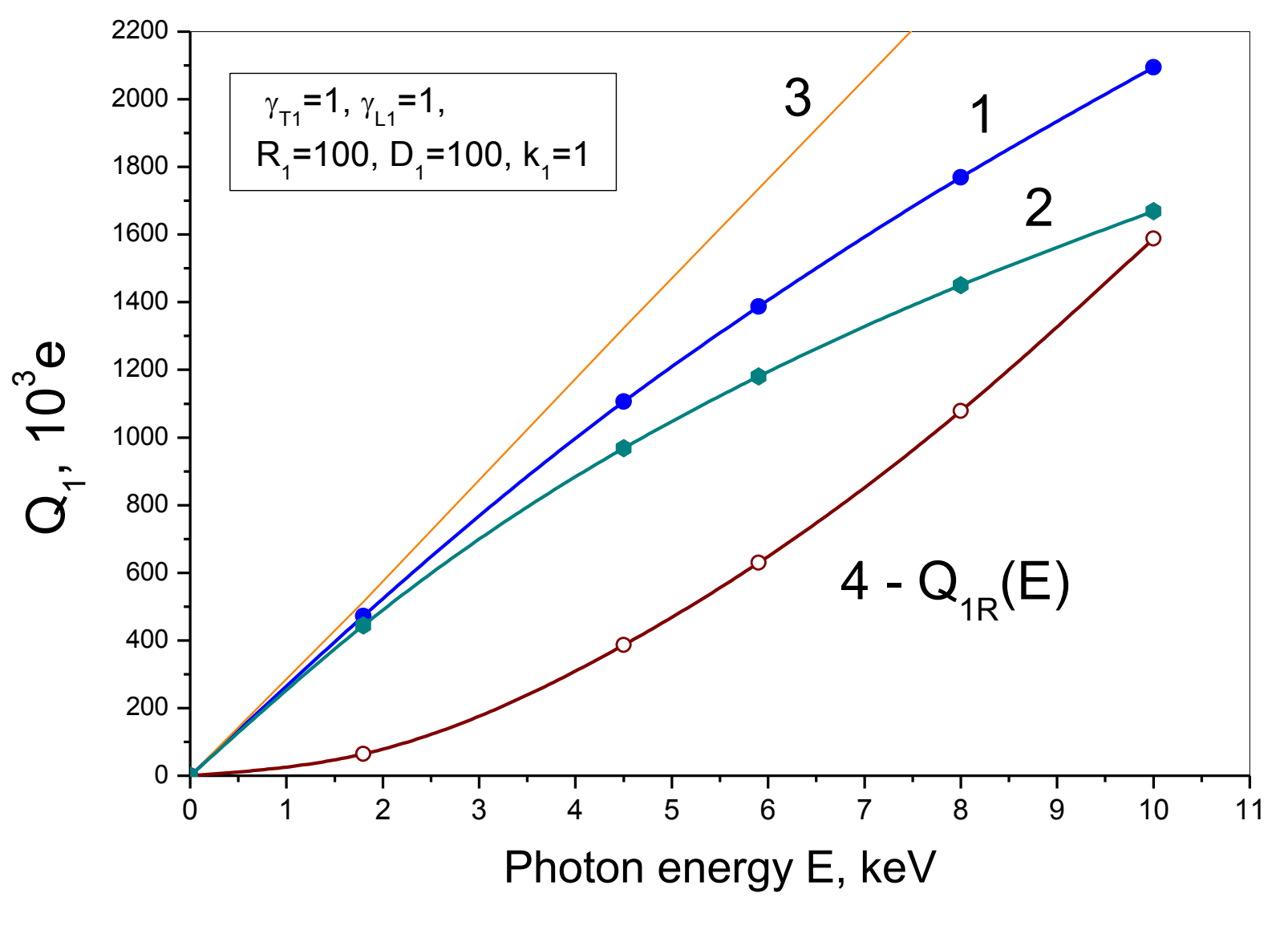
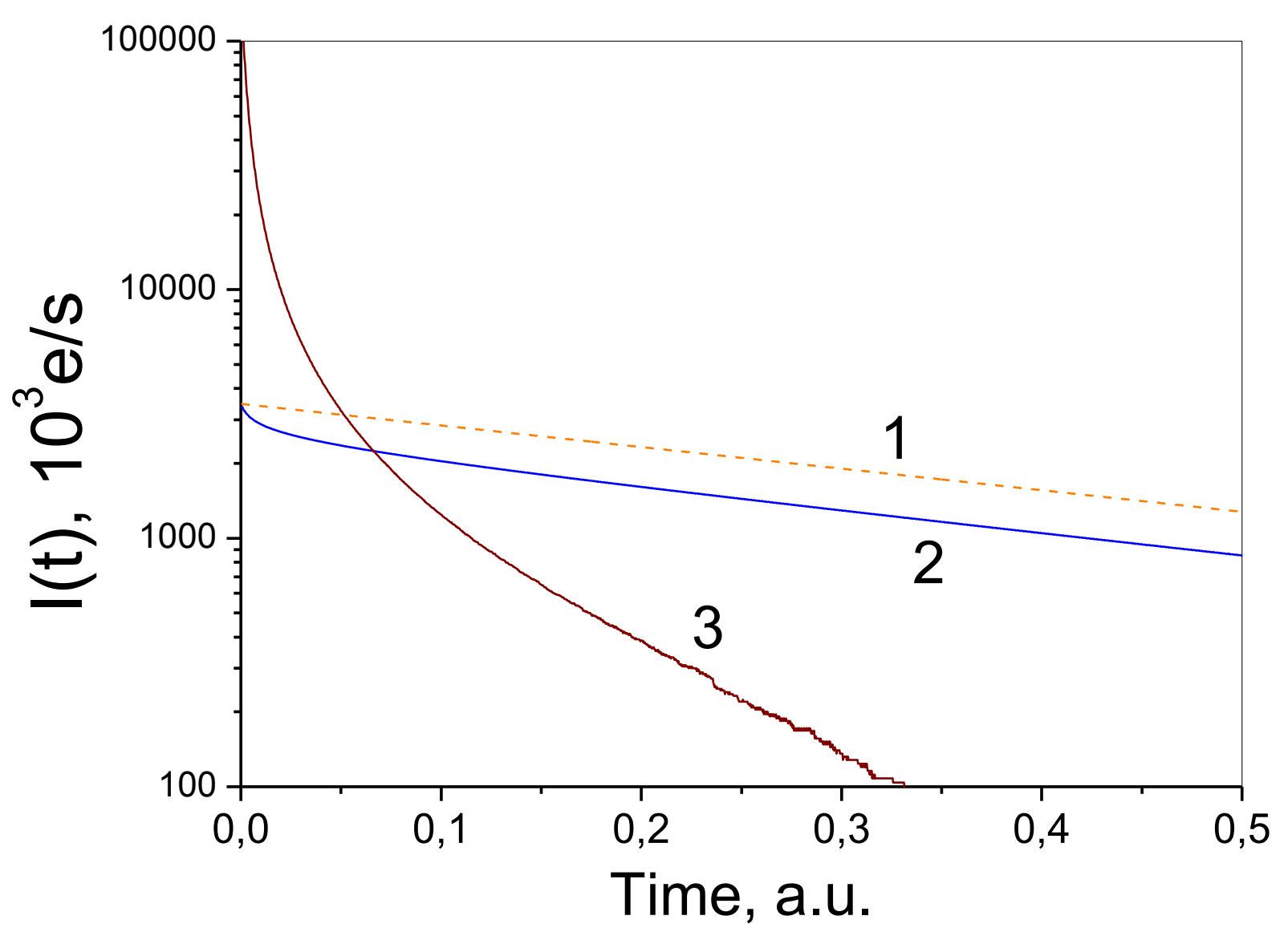


Figure 2. Dependence of detector current on time, I_1(t). 1) R=0; 2) – R=100; 3) - Time dependence of the recombination losses $I_{1R}(t)$



2. Mathematics 1

System of two diffusion equation
describing the movement of nonequilibrium quasiparticles in both electrodes of STJ detector.

$$\frac{\partial u}{\partial t} = D_1\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \gamma_{1T}u - \gamma_{1L}u + \gamma_{2T}v - R_1 u^2 + k_2 R_2 v^2$$
$$\frac{\partial v}{\partial t} = D_2\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \gamma_{2T}v - \gamma_{2L}v + \gamma_{1T}u - R_2 v^2 + k_1 R_1 u^2 \quad , \qquad (1)$$

where **u(x,y,t)** and **v(x,y,t)** are the densities of nonequilibrium quasiparticles in the first and second electrodes; ***x*** and ***y*** are coordinates ; ***t*** is time; ***D*$_1$** and ***D*$_2$** are diffusion coefficients;
γ_{1T}, γ_{1L} and **γ_{2T}, γ_{2L}** are the quasiparticle tunneling rates and loss rates in the first and second electrodes, respectively; ***R*$_1$** and ***R*$_2$** are the recombination constants; ***k*$_1$** and ***k*$_2$** coefficients of 2Δ-phonon exchange.

The initial conditions of system

$$u(x,y,t=0) = \frac{N_0}{\pi a_0^{\,2}}\exp\left(-\frac{(x-x_0)^2+(y-y_0)^2}{a_0^{\,2}}\right) \ , \quad v(x,y,t=0)=0$$

The boundary conditions:

$$D_1\frac{\partial u}{\partial n} + \alpha_1 u\Big|_B = 0 \quad , \qquad D_2\frac{\partial v}{\partial n} + \alpha_2 v\Big|_B = 0 \qquad (2)$$

The current of STJ-detector

$$I(t,x_0,y_0) = e\gamma_{1T}\iint_G u(x,y,t)dxdy + e\gamma_{2T}\iint_G v(x,y,t)dxdy = \qquad (3)$$
$$= I_1(t) + I_2(t)$$

The detector signal:

$$Q = \int_0^\infty I(t,x_0,y_0)dt = Q_1 + Q_2 \qquad (4)$$

Recombination losses *Q*$_{1R}$ (*E*)

$$Q_{1R}(E) = e\int_0^\infty\left(\iint_G R_1\left(u_1(x,y,t)\right)^2 dxdy\right)dt \quad , \qquad Q_{2R}(E) \qquad (5)$$

4. Detectors with two electrodes

In STJ detectors with two active electrodes the effect of recombination depends on a set of parameters describing properties of both detector electrodes.

Figure 3. The dependence of the signal *Q* on the photon energy in STJ-detector with parameters: $\gamma_{1T} = \gamma_{1L} = \gamma_{2T} = \gamma_{2L} = t_1 = t_2 = 1$, $D_1=D_2=100$, $R_1=R_2=100$, $k_1=1$, $k_2=0.2$.
1) – Q_1; 2) – Q_{11}; 3) – Q_{12}; 4) – Q_{1R1}; and 5) - Q_{1R2}.

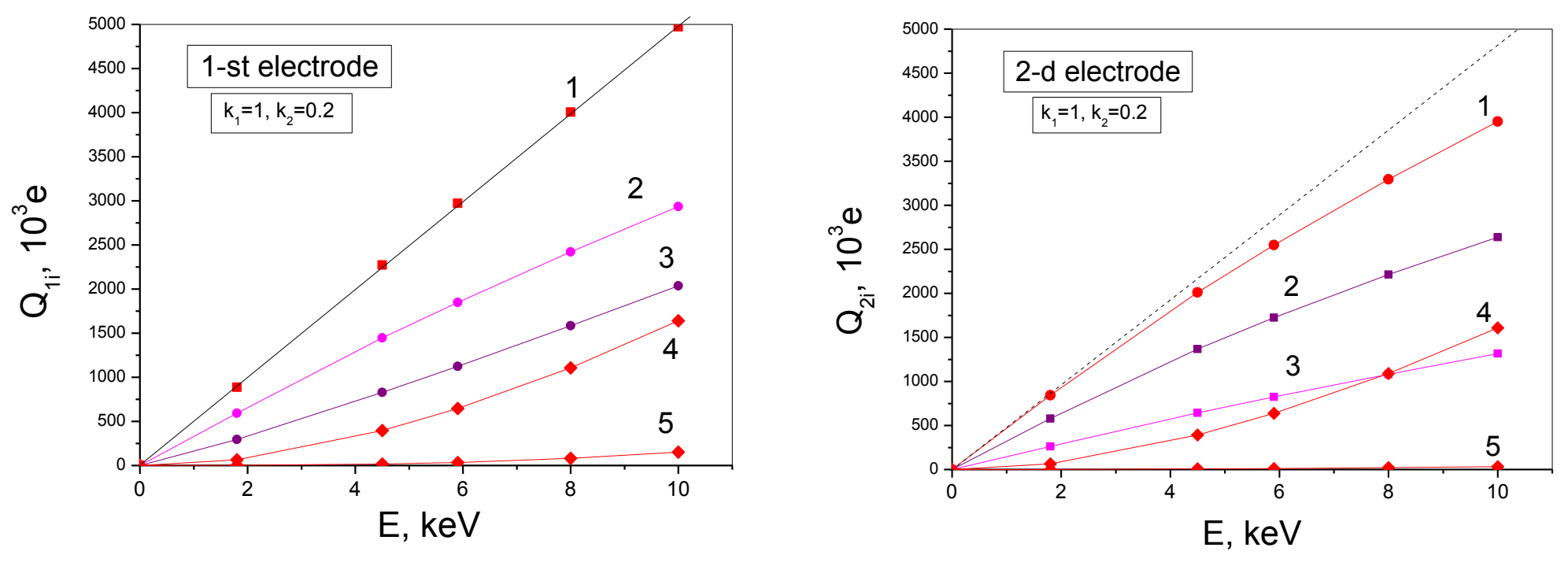


Figure 4. Dependences of the signal amplitude on the photon energy for STJ-detector with killed electrode (*Ti/Nb/Al$_2$O$_3$/Al/Nb/NbN*). Squares on curve 1 indicate experimental data Q_{top}(E) and calculations for the top electrode; the dots in curve 2 are experimental data Q_{base}(E) and calculations for the base electrode

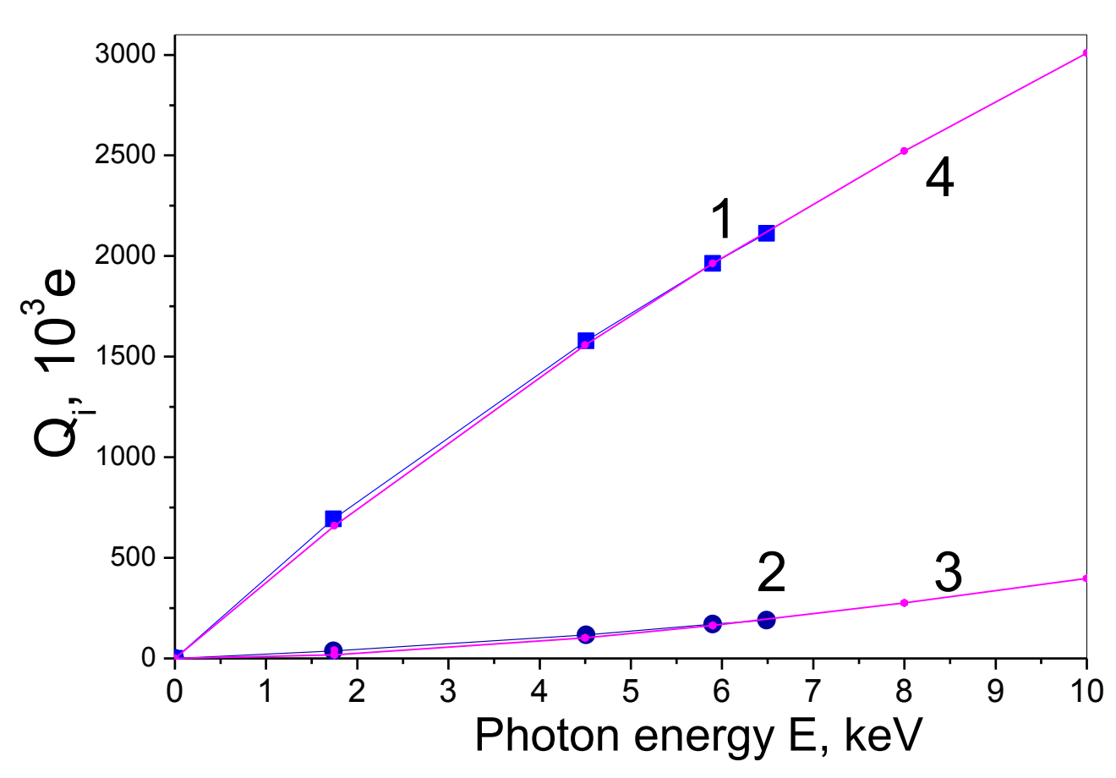
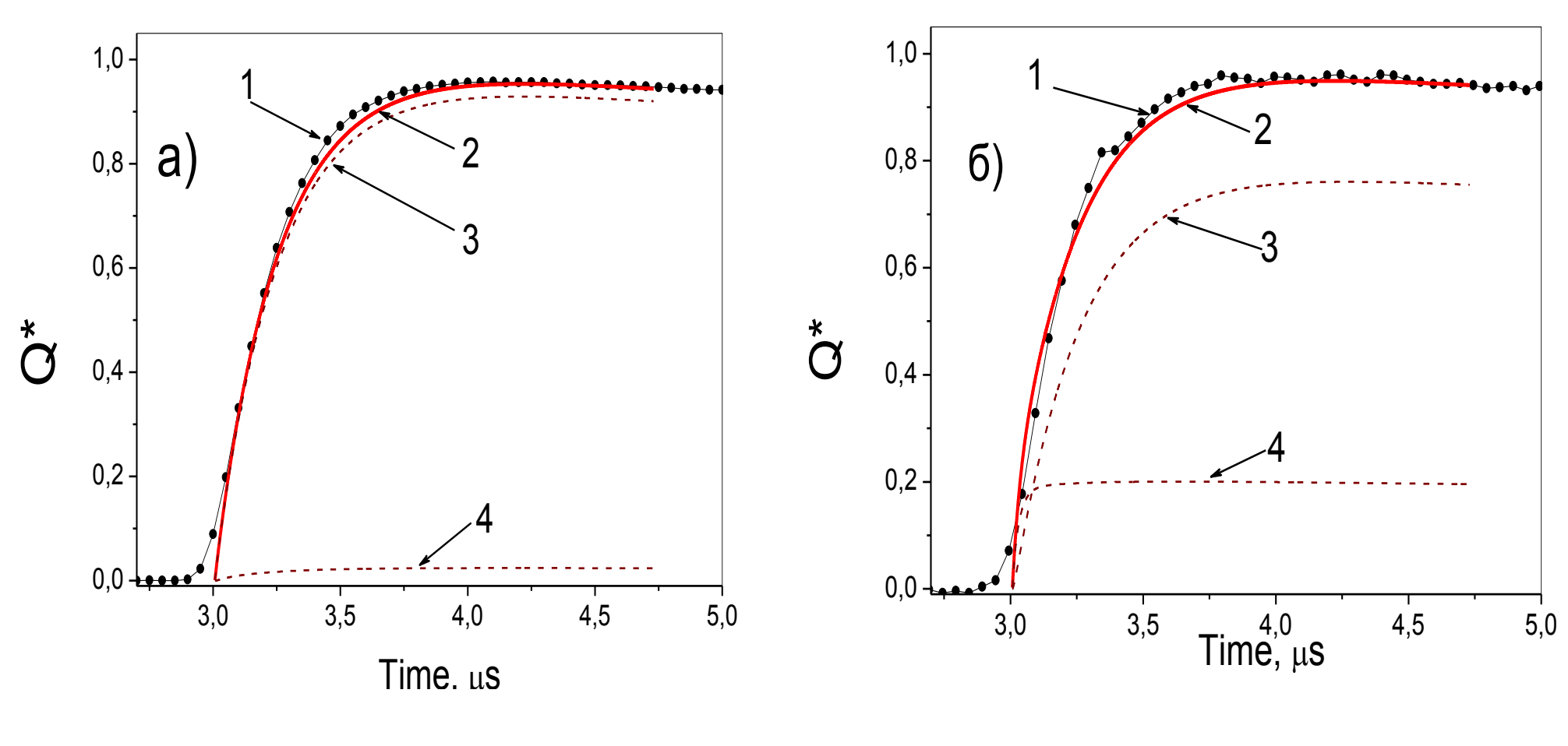


Figure 5. Time dependences of signals of the (a) top and (b) base electrodes in *Ti/Nb/Al$_2$O$_3$/Al/Nb/NbN* STJ-detector. Curves (1) correspond to the experimental Q_{top}(t) and Q_{base}(t) data. Red curves are the calculation by diffusion model.



5. Mathematics 2

Compensation of the recombination losses

$$Q(x_0,y_0) = e\cdot\int_0^\infty\left(t_1\gamma_{T1}\iint_G u(x,y,t)dxdy + e\cdot t_2\gamma_{T2}\iint_G v(x,y,t)dxdy\right)dt \qquad (6)$$

Perform the integrations over the area of the electrodes *G* and in time *t* from 0 to infinity directly in the system of differential equations (1):

$$-N_{01} = DL_1 - \gamma_{1T}\langle u\rangle - \gamma_{1L}\langle u\rangle + \gamma_{T2}\langle v\rangle - R_1\langle u^2\rangle + k_2R_2\langle v^2\rangle$$
$$0 = DL_2 - \gamma_{T2}\langle v\rangle - \gamma_{L2}\langle v\rangle + \gamma_{T1}\langle u\rangle - R_2\langle v^2\rangle + k_1R_1\langle u^2\rangle \quad , \qquad (7)$$

where we have introduced the following notation:

$$\langle u\rangle = \int_0^\infty dt\int_G u(x,y,t)dxdy \qquad \langle u^2\rangle = \int_0^\infty dt\int_G u^2(x,y,t)dxdy$$
$$\langle v\rangle = \int_0^\infty dt\int_G v(x,y,t)dxdy \qquad \langle v^2\rangle = \int_0^\infty dt\int_G v^2(x,y,t)dxdy$$

The integrals of diffusion terms are designated D_{L1} and D_{L2}. In accordance with Gauss-Ostrogradskii theorem and the expression (2), these integrals give the total losses of quasiparticles at the edges of the electrodes.
For simplicity we neglect the diffusion losses at the electrode edges, ($D_{L1} = D_{L2} = 0$), and set the coefficients t_1 and t_2 (in (6)) equal to unity.

Next, we find the solution of system (7) with respect to <u> and <v>. Then we substitute these solutions <u> and <v> in the expression for the charge (6). By simple algebraic transformations we obtain the following expression:

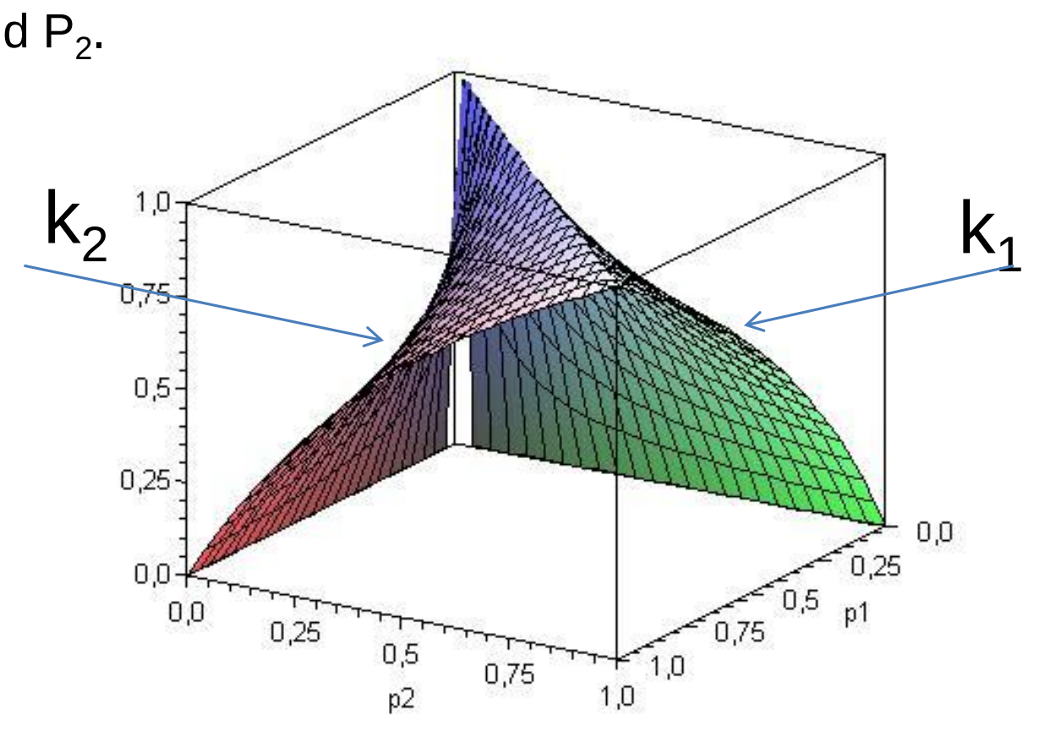
$$Q_1 = N_{01}Q_1^* - R_1\cdot\langle u^2\rangle(Q_1^* - k_1Q_2^*) - R_2\cdot\langle v^2\rangle(Q_2^* - k_2Q_1^*) \qquad (8)$$

$$Q_2^* = \frac{P_2 + P_1\cdot P_2}{1 - P_1\cdot P_2} \qquad Q_1^* = \frac{P_1 + P_1\cdot P_2}{1 - P_1\cdot P_2}$$

Conditions of compensation of the recombination:

$$Z_1 = (Q_1^* - k_1Q_2^*) = 0 \qquad Z_2 = (Q_2^* - k_2Q_1^*) = 0 \qquad (9)$$

Figure 6. Dependence of the 2Δ-phonon exchange coefficients k_1 and k_2, satisfying the conditions of compensation of recombination, as a function of P_1 and P_2.



The recombination losses may be compensated in a single electrode, but not in the both electrodes simultaneously (except $k_1 = k_2 = 1$ and Q_1=Q_2, which is not practical).

Fulfillment of compensation condition (9) only for electrode, where photon was absorbed, significantly reduces overall effect of recombination.

Figure 7. Signal amplitude Q_1 and Q_2 as function of the photon energy E. Parameters of STJ: P_1=0.59, P_2=0.7, k_1=0.9, k_2=0.5.

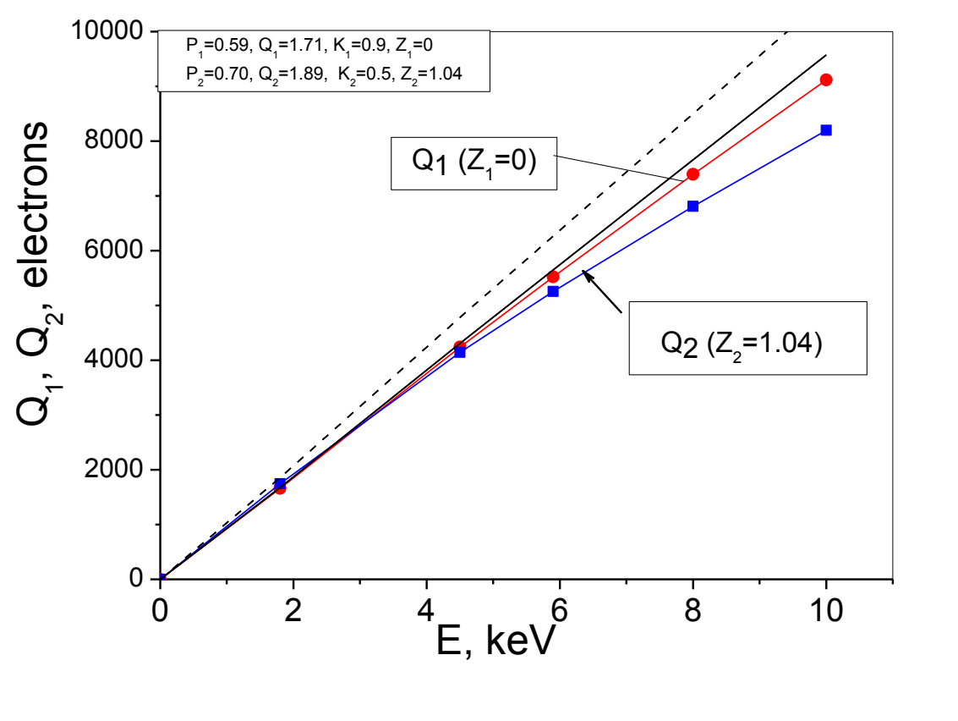
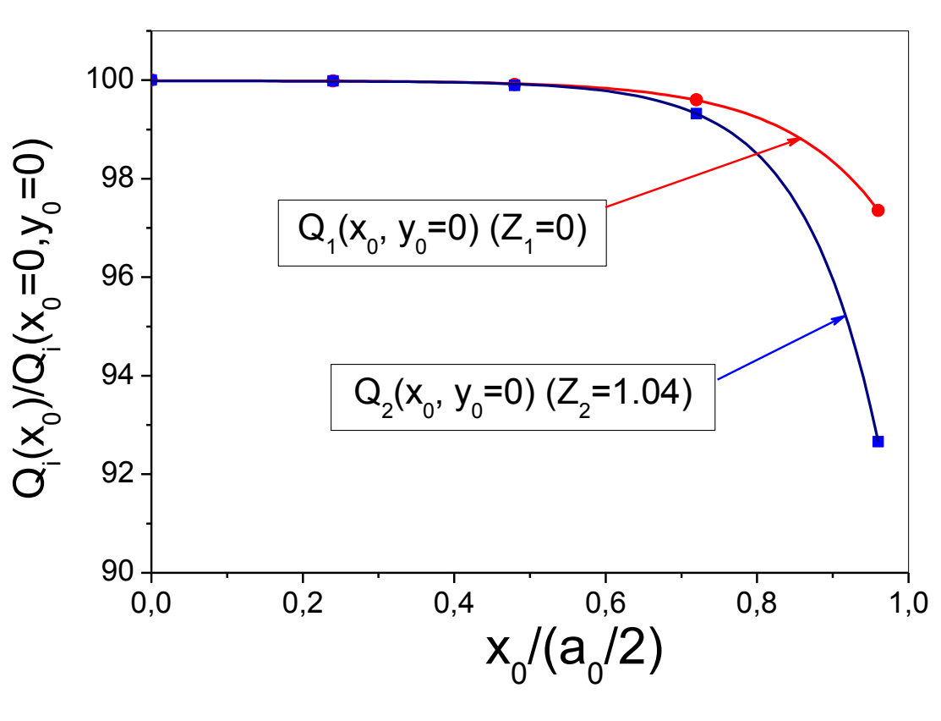


Figure 8. Dependence of the signal amplitude Q_i on the photon absorption coordinate x_0. Red – Z_1=0, Blue – Z_2=1.04



Conclusion

The effect of self-recombination is examined in detail. It was shown that the self-recombination cause: 1) a nonlinear dependence of the detector response on the photon energy; 2) broadening of the spectral line; and 3) changing in the temporal shape of the detector signals.

A new analytical expression for contribution of recombination to the signal was obtained. The conditions of the compensation of recombination losses were considered.

Noticeable weakening of recombination effects is expected in cases when the compensation condition was satisfied for the electrode, where the photon has been absorbed. In particularly, dependence of the signal amplitude on the photon energy is almost linear, and recombination broadening of the detector line is significantly weakened.